Samuel Method of Multiplication by Dividing Digits

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Abstract - Multiplication is one of four main elementary arithmetic operations. Most people learn basic multiplication algorithms in elementary school and these are used at all education levels. Among the various techniques, long multiplication is the method of multiplication that is commonly taught to elementary school students globally. It can be used on two numbers of arbitrarily large size or number of decimal digits. Long multiplication of two n-digit numbers take approximately n^2 multiplication operations. This is said to be a time complexity of order n^2 or $0(\mathrm{n}^2)$. In this paper, a new method of multiplication which works for two, three, four and five digit numbers is shown and proven. This method allows the multiplication of numbers quickly and easily in comparison to the usual method. The distribution property of multiplication over addition and subtraction are deployed; writing numbers in expanded form according to their place values is used to prove the theorems.

Index Terms - Arithmetic, place value, expanded form, digit, algorithm, theorem, complexity of order

1. Introduction

Multiplication algorithm is a method to multiply two numbers. The algorithms implemented depend on factors like the size of the numbers. Efficient multiplication algorithms have existed since the advent of the decimal system.

There is the basic form of doing multiplication tasks taught at basic schooling levels which involves long form method that involves "carrying" over different digits. However different techniques have been introduced for easy learning and assimilations. Some of these methods of multiplications are the box method, Karatsuba algorithm, The Grid system, lattice multiplication, binary multiplication, shift and add multiplication, quarter space multiplication, toom-cook multiplication, Fourier transform methods, lower bounds method, polynomial methods, Optimizing space complexity and The Trachtenberg System .

This journal provides a new method of multiplication that is easy to use for teachers and learners. There are four theorems which I have illustrated with their proofs and examples which can clearly show how to use the formulas before coming to the conclusion.

2. Theorems, Assumptions and Proofs

***Theorem 1**:*

If ab and cd are two digit numbers, their product can be obtained by the formula,

$ab \ x\ cd\ = (ab + d)(cd - d) + b\ x\ d + 10(a - c)d$, (where $ab \geq cd$).

Assumption:

If $a = c$, the formula is reduced to $(ab + d)(cd - d) + b\ x\ d$.

Proof:

$ab\ x\ cd = (ab + d)(cd - d) + b\ x\ d + 10(a - c)d$ =ab (cd-d) + d (cd-d) +b x d+10(a-c) d

$= ab\ x\ cd - ab\ x\ d + d\ x\ cd - d\ x\ d + b\ x\ d + 10a\ x\ d - 10c\ x\ d$

$= ab\ x\ cd - (10a + b)\ d + d\ (10c + d) - d\ x\ d + b\ x\ d + 10a\ x\ d - 10c\ x\ d$

$= ab\ x\ cd - 10a\ x\ d - b\ x\ d + 10c\ x\ d + d\ x\ d - d\ x\ d + b\ x\ d + 10a\ x\ d - 10c\ x\ d$

$= ab\ x\ cd - 10a\ x\ d + 10a\ x\ d - b\ x\ d + b\ x\ d + 10c\ x\ d - 10c\ x\ d + d\ x\ d - d\ x\ d$

$\therefore ab\ x\ cd = ab\ x\ cd$

Example 1: Find the outcome of $58\ x\ 57$

$58\ x\ 57$ = $(58 + 7)(57 - 7) + 8\ x\ 7$

$= 65\ x\ 50 + 56$

$= 3{,}306$

Example 2: Find the outcome of $84\ x\ 78$

$84\ x\ 78$ = $(84 + 8)\ (78 - 8)\ + 4\ x\ 8 + 10(8 - 7)\ x\ 8$

= $92x70 + 32 + 80$

$= 6{,}552$

Example 3: Find the outcome of $92\ x\ 88$

$92\ x\ 88 =\ (92 + 8)\ (88 - 8)\ + 2x8 + 10(9 - 8)\ X8$

$= 100\ x\ 80 + 16 + 10\ x\ 1\ x\ 8$

$= 8{,}000 + 16 + 10x1x8$

$= 8{,}096$

Example 4: Find the outcome of $73\ x\ 44$

$73\ x\ 44 =\ (73 + 4)\ (44 - 4)\ + 3X4 + 10(7 - 4)\ X4$

$= 77\ x\ 40 + 12 + 10\ x\ 3\ x\ 4$

$= 3{,}212$

Example 5: Find the outcome of $89\ x\ 58$

$89\ x\ 58 =\ (89 + 8)(58 - 8) + 9x8 + 10(8 - 5)x8$

$= 97x50 + 72 + 10\ x\ 3\ x\ 8$

$= 4{,}850 + 72 + 240$

$= 5{,}162$

***Theorem 2**:*

If abc and def are three digit numbers, their product can be solved by:

$abc\ x\ def = (abc + ef)(def - ef) + bc\ x\ ef + 100(a - d)x\ ef$, (Where $abc \geq$ def).

Assumption:

If $a = d$, the formula is reduced to $(abc + ef)(def - ef) + bc\ x\ ef$

Proof:

$abc\ x\ def = (abc + ef)\ (def - ef) + bc\ x\ ef + 100(a - d)\ x\ ef$

$= abc(def - ef) + ef(def - ef) + bc\ x\ ef + 100(a - d)\ x\ ef$

$= abc\ x\ def - abc\ x\ ef + ef\ x\ def - ef\ x\ ef + bc\ x\ ef + 100a\ x\ ef - 100d\ x\ ef$

$= abc\ x\ def - (100a + bc)x\ ef + ef(100d + ef) - ef\ x\ ef + bc\ x\ ef + 100a\ x\ ef - 100d\ x\ ef$

$= abc\ x\ def - 100a\ x\ ef - bc\ x\ ef + 100d\ x\ ef + ef\ x\ ef - ef\ x\ ef + bc\ x\ ef + 100a\ x\ ef - 100d\ x\ ef$

$= abc\ x\ def - 100a\ x\ ef + 100a\ x\ ef - bc\ x\ ef + bc\ x\ ef + 100d\ x\ ef - 100d\ x\ ef + ef\ x\ ef - ef\ x\ ef$

$\therefore abc\ x\ def = abc\ x\ def$

Example 1: Find the outcome of $125\ x\ 125$

$= (125 + 25)(125 - 25) + 25x25 + 100(1 - 1)x25$

$= 150x100 + 625 + 0$

$= 15{,}000 + 625$

$= 15{,}625$

Example 2: Find the outcome of $351x298$

$= (351 + 98)\ (298 - 98) + 51x98 + 100(3 - 2)x98$

$= 449x200 + 4{,}998 + 100x1x98$

$= 89{,}800 + 4{,}998 + 9{,}800$

$= 104{,}598$

Example 3: Find the outcome of $452\ x\ 377$

$= (452 + 77)\ (377 - 77) + 52x77 + 100(4 - 3)x77$

$= 529x300 + 4{,}004 + 100x1x77$

$= 158{,}700 + 4{,}004 + 7{,}700$

$= 170{,}404$

Example 4: Find the outcome of $999\ x\ 888$

$= (999 + 88)\ (888 - 88) + 99x88 + 100(9 - 8)x88$

$= 1{,}087\ x\ 800 + 8{,}712 + 100\ x\ 1\ x\ 88$

$= 869{,}600 + 8{,}712 + 8{,}800$

$= 887{,}112$

***Theorem 3**:*

If $abcd$ and $efgh$ are four digit numbers, their product can be solved by:

$abcd\ x\ efgh = (abcd + fgh)(efgh - fgh) + bcd\ x\ fgh +\ 1000(a - e)x\ fgh$

(Where $abcd \geq efgh$).

Assumption:

If $a = e$, the formula is reduced to$(abcd + fgh)(efgh - fgh) + bcdxfgh$.

Proof:

$abcd\ x\ efgh = (abcd + fgh)\ (efgh - fgh) + bcd\ x\ fgh + 1000(a - e)\ x\ fgh$

$= abcd\ (efgh - fgh) + fgh\ (efgh - fgh) + bcd\ x\ fgh + 1000\ (a - e)\ x\ fgh$

$= abcd\ x\ efgh - abcd\ x\ fgh + fgh\ x\ efgh - fgh\ x\ fgh + bcd\ x\ fgh + 1000a\ x\ fgh -$
$1000e\ x\ fgh$

$= abcd\ x\ efgh - (1000a + bcd)x\ fgh + fgh\ x\ (1000e + fgh) - fgh\ x\ fgh + bcdxfgh$
$1000a\ x\ fgh - 1000e\ x\ fgh$

$$= abcd\ x\ efgh - 1000a\ x\ fgh - bcd\ x\ fgh + 1000e\ x\ fgh + fgh\ x\ fgh - fgh\ x\ fgh + bcd\ x\ fgh + 1000a\ x\ fgh - 1000e\ x\ fgh$$

$= abcd\ x\ efgh\ - 1000a\ x\ fgh + 1000a\ x\ fgh - bcd\ x\ fgh + bcd\ x\ fgh +$
$1000e\ x\ fgh - 1000e\ x\ fgh + fgh\ x\ fgh - fgh\ x\ fgh$

$\therefore\ abcd\ x\ efgh\ =\ abcd\ x\ efgh$

Example 1: Find the outcome of $1{,}689\ x\ 1{,}456$

$=\ (1{,}689 + 456)\ (1{,}456 - 456) + 689x456 + 1000\ (1 - 1)\ x\ 456$

$= 2{,}145\ x\ 1000 + 314{,}184 + 1000\ x\ 0\ x\ 456$

$= 2{,}145{,}000 + 314{,}184 + 0$

$= 2{,}459{,}184$

Example 2: Find the outcome of $4{,}678\ x\ 3{,}175$

$= (4{,}678 + 175)\ (3{,}175 - 175) + 678\ x\ 175 + 1000\ (4 - 3)\ x\ 175$

$= 4{,}853\ x\ 3{,}000 + 118{,}650 + 1000\ x\ 1\ x\ 175$

$= 14{,}559{,}000 + 118{,}650 + 175{,}000$

$= 14{,}852{,}650$

Example 3: Find the outcome of $7{,}685\ x\ 5{,}789$

$= (7{,}685 + 789)\ (5{,}789 - 789) + 685\ x\ 789 + 1000\ (7 - 5)\ x\ 789$

$= 8{,}474\ x\ 5{,}000 + 540{,}465 + 1000\ x\ 2x\ 789$

$= 42{,}370{,}000 + 540{,}465 + 1{,}578{,}000$

$= 44{,}488{,}465$

Example 4: Find the outcome of $9{,}876\ x\ 6{,}782$

$= (9{,}876 + 782)\ (6{,}782 - 782) + 876x782 + 1000\ (9 - 6)\ x\ 782$

$= 10{,}658\ x\ 6{,}000 + 685{,}032 + 1000\ x\ 3\ x\ 782$

$= 63{,}948{,}000 + 685{,}032 + 2{,}346{,}000$

$= 66{,}979{,}032$

Theorem 4*:*

If $abcde$ and $fghij$ are five digit numbers, their product can be solved by:

$abcdexfghij = (abcde + ghij)\ (fghij - ghij) + bcde\ x\ ghij + \ 10000\ (a - f)xghij$,

(Where $abcde \geq fghij$)

Assumption:

If $a = f$, the formula is reduced to: $(abcde + ghij)\ (fghij - ghij) + bcde\ x\ ghij$

Proof:

$Abcde\ x\ fghij = (abcde + ghij)\ (fghij - ghij) + bcde\ \ x\ \ ghij + \ 10000\ (a - f)\ x\ ghij$

$= abcde\ (fghij - ghij) + ghij\ (fghij - ghij) + bcde\ \ x\ ghij + 10000\ (a - f)\ x\ ghij$

$= abcde\ x\ fghij - abcde\ x\ ghij + ghij\ x\ fghij - ghij\ x\ ghij + bcde\ x\ ghij$

$+10{,}000a\ x\ ghij - 10{,}000f\ x\ ghij$

$= abcde\ x\ fghij - (10{,}000a + bcde)x\ ghij + ghij(10{,}000f + ghij) - ghij\ x\ ghij$

ISSN 2229-5518

$+bcde\ x\ ghij\ +10{,}000a\ x\ ghij - 10{,}000f\ x\ ghij$

$= abcde\ x\ fghij - 10{,}000a\ x\ ghij - bcde\ x\ ghij\ +\ 10{,}000f\ x\ ghij\ +$

$ghij\ x\ ghij - ghij\ x\ ghij\ + bcde\ x\ ghij\ +\ 10{,}000a\ x\ ghij\ - 10{,}000f\ x\ ghij$

$= abcde\ x\ fgij - 10{,}000a\ x\ ghij\ + 10{,}000a\ x\ ghij - bcde\ x\ ghij\ + bcde\ x\ ghij$

$+10{,}000f\ x\ ghij - 10{,}000f\ x\ ghij + ghij\ x\ ghij - ghij\ x\ ghij$

$\therefore\ abcde\ x\ fghij = abcde\ x\ fghij$

Example 1: Find the outcome of $35{,}500\ x\ 35{,}500$

$= (35{,}500 + 5{,}500)\ (35{,}500 - 5{,}500) + 5{,}500x5{,}500 + 10{,}000\ (3 - 3)\ x\ 5{,}500$

$= 41{,}000\ x\ 30{,}000 + 30{,}250{,}000$

$= 1{,}230{,}000{,}000 + 30{,}250{,}000$

$= 1{,}260{,}250{,}000$

Example 2: Find the outcome of $85{,}750\ x\ 66{,}800$

$= (85{,}750 + 6{,}800)\ (66{,}800 - 6{,}800) + 5{,}750\ x\ 6{,}800 + 10{,}000\ (8 - 6)\ x\ 6{,}800$

$= 92{,}550\ x\ 60{,}000 + 39{,}100{,}000 + 136{,}000{,}000$

$= 5{,}553{,}000{,}000 + 39{,}100{,}000 + 136{,}000{,}000$

$= 5{,}728{,}100{,}000$

3. Further work

This new method for multiplication in mathematical calculations can be applied in digital settings like the computer. Hence, using the online algorithm, the multiplication procedures explained in this paper can be applicable to digital devices. .

4. Conclusion

The multiplication methods enable to multiply two, three, four and five digit numbers. The formulas stated in four theorems based on number of digits in a number with examples. It is recommended that it should be found one general formula that is applicable for all digit numbers. Hopefully, these formulas will serve as a good reference for mathematicians who want work on this area.

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Acknowledgment

First of all, I thank the almighty God who gave me the ability and strength to complete this work. I also wish to express my deep sense of gratitude and sincere thanks to Prof. Babatunde Akinola and Humphrey Adun for helping me in the process of editing.

I am grateful to my family members, Engineer Berhanu Belay,Ms. Berktit Edea,Mr. Yabets Berhanu and Miss Hadessa Berhanu, who have provided me moral and emotional support in my life.I am also grateful to other family members and facebook friends who have supported me along the way.

My gratitude also goes to Mr.Basazen and Ms. Belay, who helped me in prayer.

And finally, I appreciate the following Mojo Rafiki Village staff: Tigist Adugna, Zemen Tesfaye, Tewabech Yeshibelay and Emebet Leye .And also to Mr.Eyuel Kibebew (the Albert Einstein of our previous century) and Dr. Amanuel Leulseged for giving me continuous encouragement.

ISSN 2229-5518

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Long Life to Ethiopia!

Psalm 14

1The fool says in his heart,

“There is no God.”
They are corrupt, their deeds are vile;
there is no one who does good.
2The Lord looks down from heaven
on all mankind
to see if there are any who understand,
any who seek God.
3All have turned away, all have become corrupt;
there is no one who does good,
not even one.
4Do all these evildoers know nothing?
They devour my people as though eating bread;
they never call on the Lord.
5But there they are, overwhelmed with dread,
for God is present in the company of the righteous.
6You evildoers frustrate the plans of the poor,
but the Lord is their refuge.
7Oh, that salvation for Israel would come out of Zion!
When the Lord restores his people,
let Jacob rejoice and Israel be glad !

The Holy Bible, New International Version(NIV)